Part II:
Magnetic Resonance Imaging (MRI)

Contents

Magnetic Field Gradients
Selective Excitation
Spatially Resolved Reception
k-Space
Gradient Echo Sequence
Spin Echo Sequence
Magnetic Resonance Imaging

imaging = spatial discrimination
What nuclei can be used for imaging?

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Oxygen</td>
<td>65</td>
<td>43</td>
<td>24</td>
<td>$1.61 \times 10^{27}$</td>
</tr>
<tr>
<td>Carbon</td>
<td>18</td>
<td>16</td>
<td>12</td>
<td>$8.03 \times 10^{26}$</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>10</td>
<td>7</td>
<td>63</td>
<td>$4.22 \times 10^{27}$</td>
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<tr>
<td>Nitrogen</td>
<td>3</td>
<td>1.8</td>
<td>0.58</td>
<td>$3.9 \times 10^{25}$</td>
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<tr>
<td>Calcium</td>
<td>1.5</td>
<td>1.0</td>
<td>0.24</td>
<td>$1.6 \times 10^{25}$</td>
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<tr>
<td>Phosphorus</td>
<td>1</td>
<td>0.78</td>
<td>0.14</td>
<td>$9.6 \times 10^{24}$</td>
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<tr>
<td>Potassium</td>
<td>0.25</td>
<td>0.14</td>
<td>0.033</td>
<td>$2.2 \times 10^{24}$</td>
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<tr>
<td>Sulfur</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sodium</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chlorine</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Magnesium</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Molecules in one cell

The estimated gross molecular contents of a typical 20-micrometre human cell is as follows:[5]

<table>
<thead>
<tr>
<th>Molecule</th>
<th>Percent of Mass</th>
<th>Mol. Weight (daltons)</th>
<th>Molecules</th>
<th>Percent of Molecules</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>65</td>
<td>18</td>
<td>$1.74 \times 10^{14}$</td>
<td>98.73</td>
</tr>
<tr>
<td>Other Inorganics</td>
<td>1.5</td>
<td>55</td>
<td>$1.31 \times 10^{12}$</td>
<td>0.74</td>
</tr>
<tr>
<td>Lipids</td>
<td>12</td>
<td>700</td>
<td>$8.4 \times 10^{11}$</td>
<td>0.475</td>
</tr>
<tr>
<td>Other Organics</td>
<td>0.4</td>
<td>250</td>
<td>$7.7 \times 10^{10}$</td>
<td>0.044</td>
</tr>
<tr>
<td>Protein</td>
<td>20</td>
<td>50,000</td>
<td>$1.9 \times 10^{10}$</td>
<td>0.011</td>
</tr>
<tr>
<td>RNA</td>
<td>1.0</td>
<td>$1 \times 10^6$</td>
<td>$5 \times 10^7$</td>
<td>3 x 10^-5</td>
</tr>
<tr>
<td>DNA</td>
<td>0.1</td>
<td>$1 \times 10^{11}$</td>
<td>46</td>
<td>3 x 10^-11</td>
</tr>
</tbody>
</table>

adapted from wikipedia: http://en.wikipedia.org/wiki/Composition_of_the_human_body
For conventional imaging (x-ray) imaging properties are given by the geometry of the light path.
Frequency range for MRI: 10-400 MHz
Wavelength: 63.87 MHz ~ 470cm
Magnetic Resonance Imaging

Objects that are smaller than the wave length cannot be imaged (geometric imaging not possible!)
Magnetic Resonance Imaging

Sample in a homogenous magnetic field $B_0$

All magnetic moments precess with the same frequency, i.e., with the Larmor frequency.
Magnetic Resonance Imaging: Gradients

Switchable, linear magnetic field gradients independently in x, y and z direction

Gradients are used to give protons different Larmor frequencies at different positions!

\[ \mathbf{v}(\mathbf{r}) = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \gamma (B_0 \mathbf{e}_z + \mathbf{G} \cdot \mathbf{r}) = \gamma \begin{bmatrix} G_x x \\ G_y y \\ B_0 + G_z z \end{bmatrix} \]
Magnetic Resonance Imaging: Gradients

Sample in a homogenous magnetic field $B_0$ with gradient field $G_z$

There is a 1:1 correspondence between the frequency and the position!
Magnetic Resonance Imaging: Slice Selection

Frequency of $B_1$ field is $\omega = \gamma B_0$ (resonance condition)

Result: 90 degrees pulse!
Magnetic Resonance Imaging: Slice Selection

Frequency of $B_1$ field is $\omega > \gamma B_0$  (off resonance condition)

Result: 32 degrees pulse!
Magnetic Resonance Imaging: Slice Selection

Frequency of $B_1$ field is $\omega >> \gamma B_0$ (off resonance condition)

Result: 1 degrees pulse!
Effective B1 depends on frequency of B1 field:

\[
\begin{align*}
\left[ \frac{d\vec{M}}{dt} \right]_{rot} &= \gamma \vec{M} \times \vec{B}_{eff} \\
\vec{B}_{eff} &= \begin{pmatrix} B_1 \\ 0 \\ B_0 - \frac{\omega_1}{\gamma} \end{pmatrix}
\end{align*}
\]
Magnetic Resonance Imaging: Slice Selection

RF

$G_z$

Excitation profile

Signal only from a selected volume/slice

This is not so nice, but we are close!
Magnetic Resonance Imaging: Slice Selection

Excitation profile

Signal only from a selected volume/slice

RF

G_z

B(z) = γGz

ω_0

z

ω_0

90°

flip angle

B_0
Magnetic Resonance Imaging: Slice Selection

Position (z)

$B_0 + G_z z$

$B_0$

frequency

flip angle

90

$\omega_0$

frequency
Magnetic Resonance Imaging: Spatially Resolved Reception

RF

Gz

Rx

Z

X

RF

Gz

Rx
Magnetic Resonance Imaging: Spatially Resolved Reception

$S(t)$

Gradient $G_x$

$S(\omega)$

$\omega \approx x$

$\omega_1$

$\omega_2$

$x_1$

$x_2$
Magnetic Resonance Imaging: Spatially Resolved Reception

RF

$G_z$

$G_x$

ADC

$S(\omega)$

Gradient $G_x$

$\omega_1$

$\omega_2$

$x_2$

$x_1$

1D FFT

Back projection
Magnetic Resonance Imaging: Spatially Resolved Reception

RF

$G_z$

$G_x$

$G_y$

ADC

1D FFT

Back projection
Magnetic Resonance Imaging: Spatially Resolved Reception

RF

$G_z$

$G_x$

$G_y$

ADC

[Diagram showing the sequence of RF, G, and ADC signals with a brain image on the right side.]
Magnetic Resonance Imaging: Spatially Resolved Reception

RF  
\[ G_z \]  
\[ G_x \]  
\[ G_y \]  
ADC
Magnetic Resonance Imaging: Spatially Resolved Reception

RF

$G_z$

$G_x$

$G_y$

ADC

[Diagram of magnetic resonance imaging pulses and signal]
Magnetic Resonance Imaging: Spatially Resolved Reception

RF

$G_z$

$G_x$

$G_y$

ADC

[Diagram of RF pulse, gradient fields ($G_z$, $G_x$, $G_y$), and ADC signal with an image of a brain scan]
Magnetic Resonance Imaging: Spatially Resolved Reception

**RF**

**G_z**

**G_x**

**G_y**

**ADC**
Magnetic Resonance Imaging: Spatially Resolved Reception

RF

$G_z$

$G_x$

$G_y$

ADC
Magnetic Resonance Imaging: Spatially Resolved Reception

RF
G_z
G_x
G_y
ADC
Magnetic Resonance Imaging: Spatially Resolved Reception

Repeat $n$ times along different angles
Magnetic Resonance Imaging: Spatially Resolved Reception

Repeat $n$ times along different angles
Magnetic Resonance Imaging: Imaging in k-Space

A static point source at some position $\mathbf{r}$ (e.g., drop of water):

$$S_r(t) = S_r(0) e^{i\omega t}$$

frequency $\omega$ is a function of time (in the presence of gradients $\mathbf{G}$):

$$\omega(t) = \gamma \mathbf{G}(t) \cdot \mathbf{r} \quad \Rightarrow \quad S(t) = S(0) e^{i \gamma \int_0^t \mathbf{G}(t) \cdot \mathbf{r} \, dt'}$$

the total signal from an object with given nuclei density $\rho(\mathbf{r})$:

$$S(t) = \int \rho(\mathbf{r}) e^{i \gamma \int_0^t \mathbf{G}(t) \cdot \mathbf{r} \, dt'} \, d\mathbf{r}$$
Magnetic Resonance Imaging: Imaging in k-Space

The total signal from an object with given nuclei density \( \rho(r) \):

\[
S(t) = \int \rho(r) e^{i \gamma \int_0^t G(t') \cdot r \, dt'} \, dr
\]

\[
k(t) := \frac{1}{2\pi} \gamma \int_0^t G(t) \, dt
\]

With the definition of the wave-vector \( k \equiv k(t) \), we get for the total signal:

\[
S(k) = \int \rho(r) e^{i 2\pi k \cdot r} \, dr
\]

and after inversion of the signal equation:

\[
\rho(r) = \int S(k) e^{-i 2\pi k \cdot r} \, dk
\]
Magnetic Resonance Imaging: Imaging in k-Space

\[ S(k) \equiv S(k(t)) = S(t) \]

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ \rho(r) = \int S(k) e^{-i2\pi k \cdot r} \, dk \]
Magnetic Resonance Imaging: k-Space Properties
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Magnetic Resonance Imaging: k-Space Properties
Magnetic Resonance Imaging: k-Space Properties
Magnetic Resonance Imaging: k-Space Properties

FOV in x-direction

FOV in y-direction

resolution in y-direction

resolution in x-direction
Magnetic Resonance Imaging: k-Space & Sequences

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ S(k) \equiv S(k(t)) = S(t) \]

\[ k(t) := \frac{1}{2\pi} \gamma \int_0^t G(t) \, dt \]
Magnetic Resonance Imaging: Gradient Echo (GRE)

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ S(k) \equiv S(k(t)) = S(t) \]

\[ k(t) := \frac{1}{2\pi} \gamma \int_0^t G(t) \, dt \]
Magnetic Resonance Imaging: Gradient Echo (GRE)

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ S(k) \equiv S(k(t)) = S(t) \]

\[ k(t) := \frac{1}{2\pi} \gamma \int_0^t G(t) \, dt \]
Magnetic Resonance Imaging: Gradient Echo (GRE)

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ S(k) \equiv S(k(t)) = S(t) \]

\[ k(t) := \frac{1}{2\pi} \gamma \int_0^t G(t) \, dt \]
Magnetic Resonance Imaging: Gradient Echo (GRE)

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ S(k) \equiv S(k(t)) = S(t) \]

\[ k(t) := \frac{1}{2\pi} \gamma \int_0^t G(t) \, dt \]
Magnetic Resonance Imaging: Gradient Echo (GRE)

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ S(k) = S(k(t)) = S(t) \]

\[ k(t) := \frac{1}{2\pi} \gamma \int_0^t G(t) \, dt \]
Magnetic Resonance Imaging: Gradient Echo (GRE)

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ S(k) \equiv S(k(t)) = S(t) \]

\[ k(t) := \frac{1}{2\pi} \gamma \int_0^t G(t) \, dt \]
Magnetic Resonance Imaging: Gradient Echo (GRE)

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ S(k) \equiv S(k(t)) = S(t) \]

\[ k(t) := \frac{1}{2\pi} \gamma \int_0^t G(t) \, dt \]
Magnetic Resonance Imaging: Echo Planar Imaging (EPI)

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ S(k) \equiv S(k(t)) = S(t) \]

\[ k(t) := \frac{1}{2\pi} \gamma \int_{0}^{t} G(t) \, dt \]
Magnetic Resonance Imaging: Spiral EPI

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ S(k) \equiv S(k(t)) = S(t) \]

\[ k(t) := \frac{1}{2\pi} \gamma \int_0^t G(t) \, dt \]
Magnetic Resonance Imaging: Spin Echo (SE)

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ S(k) \equiv S(k(t)) = S(t) \]

\[ k(t) := \frac{1}{2\pi} \gamma \int_0^t G(t) \, dt \]
Magnetic Resonance Imaging: Spin Echo (SE)

\[ S(k) = \int \rho(r) e^{i2\pi k \cdot r} \, dr \]

\[ S(k) \equiv S(k(t)) = S(t) \]

\[ k(t) := \frac{1}{2\pi} \gamma \int_0^t G(t) \, dt \]
Summary: Part II

Gradients induce a linear change in magnetic fields along one spatial direction.

As a result, gradients induce a linear change in the larmor frequency of the magnetization with position (related by the Fourier transform).

In combination with an exitation field, slice selection can be achieved.

Using back transform (this is CT!), the spatial dependency in the precession frequency can be used to generate an image.

The variation in the spatial frequency (and phase) from switching gradients in different orthogonal directions is stored in k-space.

Fourier transform related k-space with image space.

Gradient echoes are recalled with gradients, spin-echoes using a 180° pulse.